Quantifying Gerrymandering With Simulated Annealing: A Study of Texas

Stuart Wayland | Faculty Advisor: Professor Eric Vigoda

What is Gerrymandering?

Definition When political or electoral districts are drawn with the purpose of giving one political group an advantage over another, a practice which often results in districts with bizarre or strange shapes (Cornell Law)

Motivation Unfair districtings can produce immensely disproportional election results (see Figure 1)

An MCMC Solution

Goal We want to accurately quantify the fairness of a districting plan

Solution

1. Use a Markov Chain Monte Carlo to generate random districting plans
2. Compare the election outcome of an existing plan to the distribution of randomly generated plans
3. Set fairness of a districting plan as the probability of producing the same election result

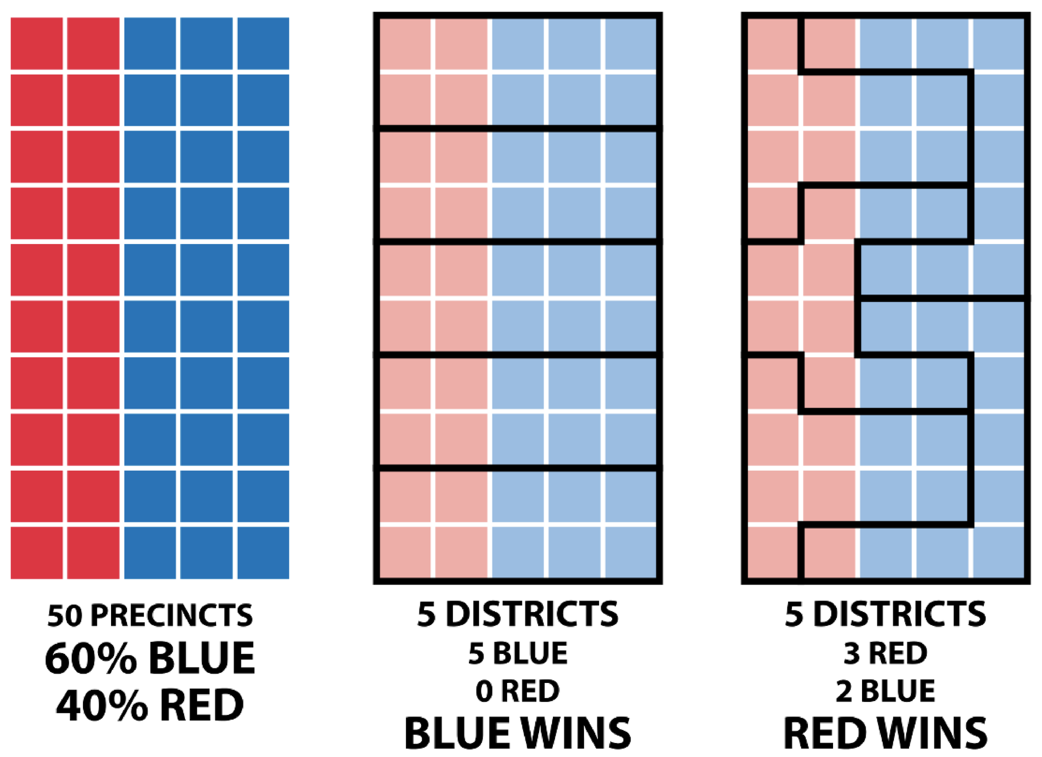

Figure 1. Example Districts with Election Outcomes

Districting as a Graph Coloring Problem

Generating a Dual Taking a state shapefile, we generate the dual of the shapefile

N-coloring An n-coloring of the dual graph represents a possible districting, where n is the number of districts

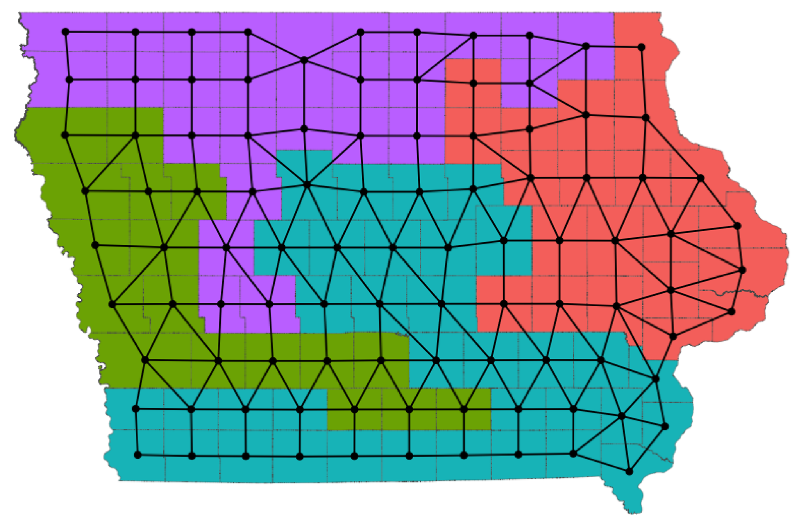

Figure 2. Coloring of Iowa County Shapefile with Dual

Transitions in the MCMC Algorithm

Markov Chain Monte Carlo performs a random walk on the space of all valid n-colorings

Simple Transition

1. Choose a vertex and change its color
2. If a valid n-coloring, transition with a preset probability p

Viable n-colorings

Contiguous each color must be a connected component (invalid if not)

Population Equality minimize population difference between colors (population score)

Compactness minimize number of edges between colors (compactness score)

Simulated Annealing

Score Function With weights β_1, β_2,

$$w(\text{coloring}) = e^{-\beta_1(\text{population score}) - \beta_2(\text{compactness score})}$$

Metropolis Filter

$$p = \min\left\{1, \frac{w(\text{new coloring})}{w(\text{old coloring})}\right\}$$

Transition probability sets sampling distribution proportional to weight function

Simulated Annealing

Start with small β_1, β_2 so chain moves freely over space of valid n-colorings

Increase β_1, β_2 to return to viable n-colorings

Results in Texas

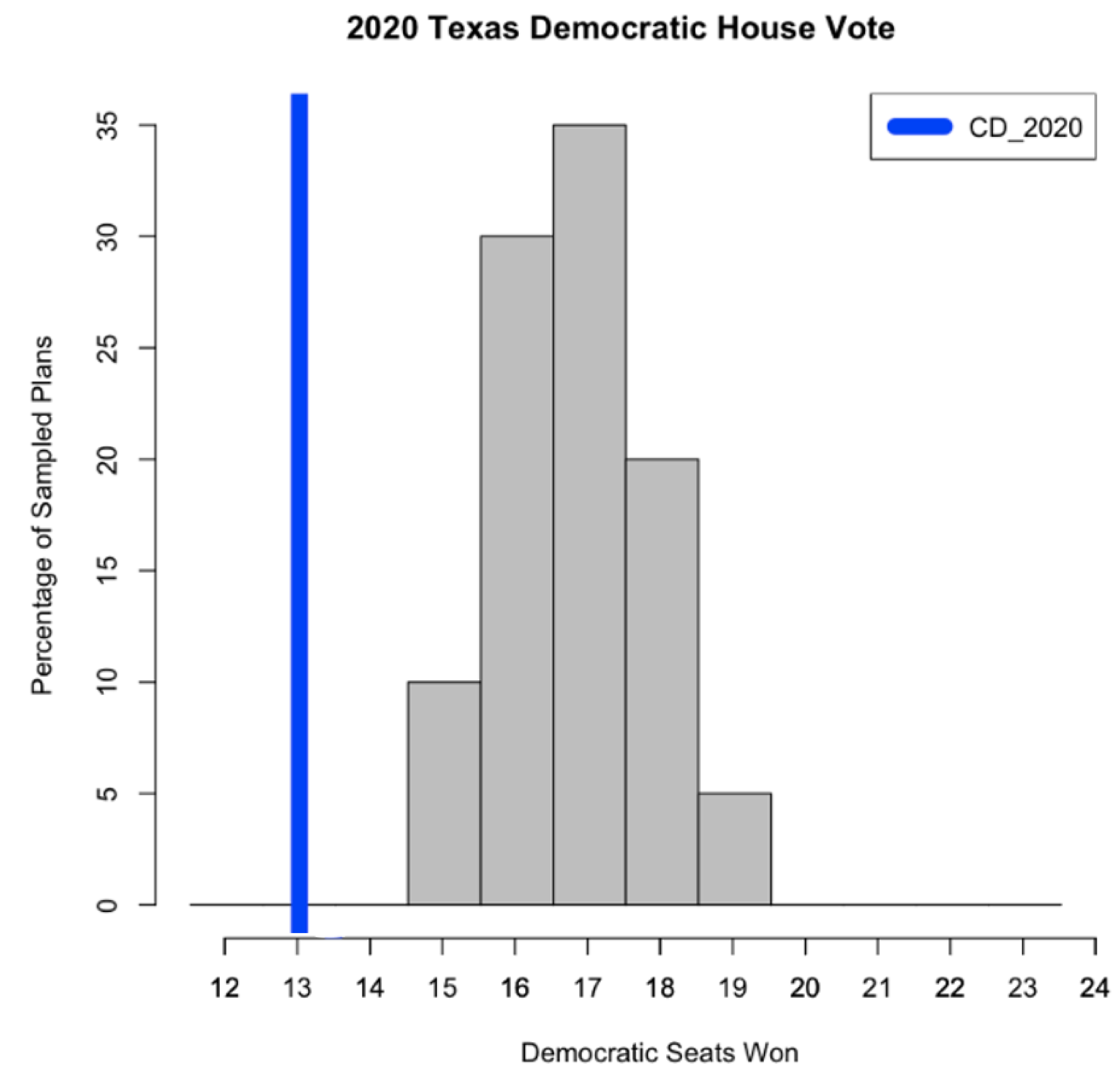

Figure 3. 2020 TX US House Election Distribution

Conclusion We have no sampled plans with 13 (as in current plan) or even 14 Democratic seats

Department of Computer Science

University of California, Santa Barbara–June 3, 2020

stuartwayland@ucsb.edu